Low-lying resonances of ${}^{9}_{\Lambda}$Be: Faddeev calculation with Pade-approximants

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Configuration space Faddeev equations are applied to describe the ${}^{9}_{\Lambda}$Be hypernucleus in the $\alpha\alpha\Lambda$ cluster model. For calculation of resonance state energies a variant of the method of analytical continuation in coupling constant is used. To realize this method, an auxiliary three-body potential is added to the Hamiltonian of the equations. A proper choice of strength parameter of the potential converts a resonance state into a bound state and an energy trajectory is obtained by variations of this parameter. To fulfill analytical continuation of this trajectory as a function of the strength parameter onto the complex plane, the Pade' approximation is used. Spectrum of low-lying resonances is calculated with two $\alpha\Lambda$ phenomenological potentials. We predict existence of the 0^+_2 and 4^+_1 virtual states as well as the 2^+_2 resonance state near by the $\alpha\alpha\Lambda$ threshold. The ^{8}Be(L^+)+$\Lambda(s$-wave) configuration for description of the ${}^{9}_{\Lambda}$Be ground band is discussed.

1. Introduction

Recently a new interest appeared to the method of analytical continuation in coupling constant. The method was suggested [1] a long time ago to calculate resonance state parameters of a three-body system. Though this method is based on approximate continuation onto the complex plane using a number of negative energies, enough accurate results were obtained in [2]. In that paper the analytical continuation was applied combined with complex scaling method. In paper [3], it was also shown that this method allows one to obtain accurate calculations of low-lying resonances.

In the present work we applied a variation of the method of analytical continuation in coupling constant for calculations of energy of resonances using the $\alpha+\alpha+\Lambda$ cluster model [4] for the ${}^{9}_{\Lambda}$Be hypernucleus. In this variant of the method an additional non-physical three-body potential is defined. To perform the analytical continuation, we use a number of negative energy values obtained by changing the strength parameter (coupling constant) of this additional potential. The unchanging of the pair interactions and two-body thresholds is an important attribute of such an approach. Calculations of bound state energies are performed on the basis of the Faddeev equations in the configuration

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space. To describe $\alpha\alpha$ nuclear interaction, the Ali-Bodmer potential [7] having s, d and g-wave components is used. Spectra of low-lying resonances are calculated with two $\alpha\Lambda$ phenomenological potentials. We predict the existence of a 2^+ resonance state close to the $\alpha\alpha\Lambda$ threshold. The classification of the ${}^{9}_{\Lambda}$Be ground band levels which was given in [4,5,6] is discussed.

2. Model and method

Coupled set of the Faddeev equations describing the ${}^{9}_{\Lambda}$Be nucleus in the $\alpha\alpha\Lambda$ cluster model is written in the form:

$$\begin{array}{l}(H_0 + V^s_{\alpha\Lambda} + V^c - E)W = -V^s_{\alpha\Lambda}(U - P_{12}W),\\ (H_0 + V^s_{\alpha\alpha} + V^c - E)U = -V^s_{\alpha\alpha}(W - P_{12}W),\end{array} \qquad (1)$$

where H_0 is the kinetic energy operator, P_{12} is the permutation operator for the α particles (particles 1,2), $V^s_{\alpha\alpha}$ and $V^s_{\alpha\Lambda}$ are nuclear potentials of $\alpha\alpha$ and $\alpha\Lambda$ interactions, respectively. V^c is the potential of the Coulomb interaction between α particles, U is the Faddeev component corresponding to the rearrangement channel $(\alpha\alpha) - \Lambda$ and W corresponds to the rearrangement channel $(\alpha\Lambda) - \alpha$. The total wave function is expressed via the components U and W: $\Psi = U + (1 - P_{12})W$. The total orbital angular momentum is given by $\vec{L} = \vec{\ell}_{\alpha\alpha} + \vec{\lambda}_{(\alpha\alpha)-\Lambda} = \vec{\ell}_{\alpha\Lambda} + \vec{\lambda}_{(\alpha\Lambda)-\alpha}$, where $\ell_{\alpha\alpha}$ ($\ell_{\alpha\Lambda}$) is the orbital angular momentum of the α's (pair of $\alpha\Lambda$) and $\lambda_{(\alpha\alpha)-\Lambda}$ ($\lambda_{(\alpha\Lambda)-\alpha}$) is the orbital angular momentum of a Λ hyperon (α particle) relative to the center of mass of the pair α ($\alpha\Lambda$) particles. More detailed description of this formalism may be found in ref. [8]. To describe interactions in the $\alpha\alpha\Lambda$ system, local pairwise potentials were used. The nuclear $\alpha\alpha$ interaction is given by version "a" of the phenomenological Ali-Bodmer potential [7] having s, d and g - wave components, which was modified in work [9]. For $\alpha\Lambda$ interaction, phenomenological potentials having the form of one (Gibson [10]) and two rank (Isle [11]) Gaussian were used. Each potential have the same components as s-wave component in all partial waves of the $\alpha\Lambda$ pair. To estimate the energies of low-lying resonance states, we applied a method [2,3] which is based on an analytical continuation with strength parameter for additional three-body potential (coupling constant). This continuation in the unbound state region is carried out using the Padé approximants. The three-body potential having the form: $V_3(\rho) = -\delta \exp(-b\rho^2)$ is added in the left hand sides of Eqs. (1). Parameter of this potential b=0.1fm^{-1}, $\delta \geq 0$ is variational parameter, and $\rho^2 = x^2_\alpha + y^2_\alpha$ where x_α, y_α are the scaled Jacobi coordinates ($\alpha = 1, 2$) [3]. For each resonance there exists the region $|\delta| \geq |\delta_0|$ where a resonance is by a bound state. The three-body bound state calculations were performed using $(2N)$ values of δ. The continuation onto complex plane is carried out by means of the Padé approximant: $\sqrt{-E} = \frac{\sum_{i=1}^{N} p_i \xi^i}{1+\sum_{i=1}^{N} q_i \xi^i}$ where $\xi = \sqrt{\delta_0 - \delta}$. The Padé approximant for $\delta = 0$ gives the energy and width of resonance: $E(\delta = 0) = E_r + i\Gamma/2$. Note that accuracy of the Padé approximation for resonance energy and width decreases with increasing of distance from scattering threshold.

3. Calculations

The coupled configuration space Faddeev equations (1) for the $\alpha\alpha\Lambda$ system are solved numerically using the finite-difference method and spline appoximation [8,12]. In Table

1 we present the 2_2^+ resonance parameters calculated with Pade' approximation of various orders. Accuracy of our calculations is defined by the decimal digit. Pade' approximant as a function of the "coupling constant" δ is shown in Figure 1 together with values of bound state energies used for construction of the approximants. The calculated energies of the resonances (2_2^+, 4_2^+), bound (0_1^+, 2_1^+) and virtual (0_1^+, 4_1^+) states for ${}^9_\Lambda$Be ground band are shown in Figure 2 for the case of the Gibson $\alpha\Lambda$ potential. For the Isle potential, the localization of low-lying levels is similar to one showed in Figure 2, though the potential overbinds the ground state for 2 MeV, approximately [8]. Results for Isle potential are shifted downwards relative to corresponding Gibson potential results for the 0_1^+, 2_1^+, 4_2^+ states. In Figure 2, we show also the experimental data for ^{8}Be ground band [13]. In ref. [4,5,6] ground band of ${}^9_\Lambda$Be was classified as an analog of the ^{8}Be ground band. According to this classification ${}^9_\Lambda$Be is considered as the ^{8}Be core plus s-wave hyperon: ${}^8\mathrm{Be}(\mathrm{L}^+)+\Lambda(s\text{-wave})$, when $L = 0, 2, 4$. As is seen from Figure 2, the results of our calculation do not agree with such simple scheme. In these calculations we used two sets of subsystem momenta l and λ to evaluate contributions in energy due to higher partial waves. The minimal configurations of orbital momenta ($l_{\alpha\alpha}$,$\lambda_{(\alpha\alpha)-\Lambda}$) corresponding to the states 0^+, 2^+, 4^+ are {(0,0)}, {(0,2),(2,0)}, {(0,4),(4,0)}, respectively. At the same time in the $(\alpha\Lambda)-\alpha$ channel only the first configurations with minimal values of momenta are used. Corresponding results in Figure 2 are called "minimal" ones. Note that from Figure 2 one could conclude that other possible orbital momenta configurations play an important role in formation of these states. Higher partial waves taken into account in orbital momenta configurations shift the energy levels. In Figure 2 these configurations are called "maximal" ones. They include six first minimal successive values of momenta for both $(\alpha\alpha)-\Lambda$ and $(\alpha\Lambda)-\alpha$ channels. We have found a new 2_2^+ resonance state that is near by the $\alpha\alpha\Lambda$ threshold. In addition the 0_2^+ and 4_1^+ states are appeared as virtual states. From Figure 2 one can see the 4_1^+ state, which is a resonance state in minimal configuration, becomes a virtual state in maximal configuration. The 4^+ resonance of ${}^9_\Lambda$Be is appeared as the second 4^+ state.

Table 1
Calculated 2_2^+ energy E_r and width $\Gamma/2$ ($E_r + i\Gamma/2$) (in MeV) in dependence on the order N of the Pade' approximant with δ_0 = -4.0 MeV (-4.62 MeV) for Gibson (Isle) potential. The energy is measured from the ${}^5_\Lambda$He+α threshold.

N	Gibson	Isle
2	3.8(0)+i4.3(2)	3.2(0)+i3.2(6)
3	3.8(2)+i4.3(2)	–
4	3.8(3)+i4.3(2)	3.0(9)+i3.2(6)

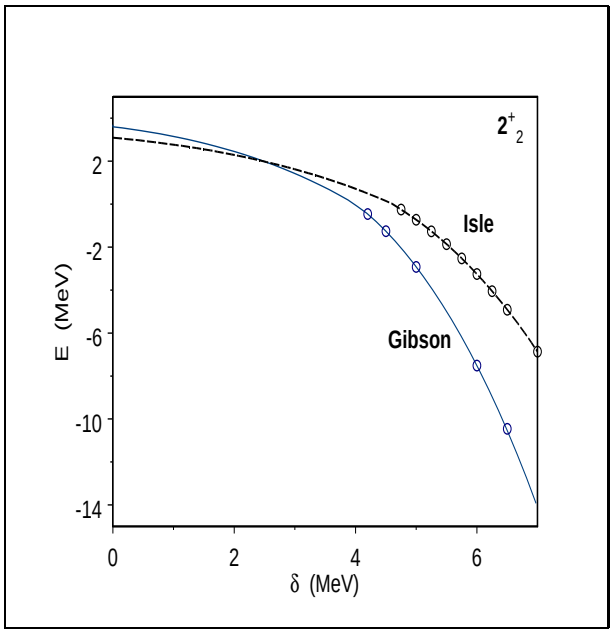

Figure 1. The Pade' approximants for 2_2^+ resonance of the $\alpha\alpha\Lambda$ system. The solid (dashed) line corresponds to calculation with Gibson(Isle) $\alpha\Lambda$ potential. Calculated bound state energies are shown by circles. Energies are measured from the ${}^5_\Lambda$He+α threshold.

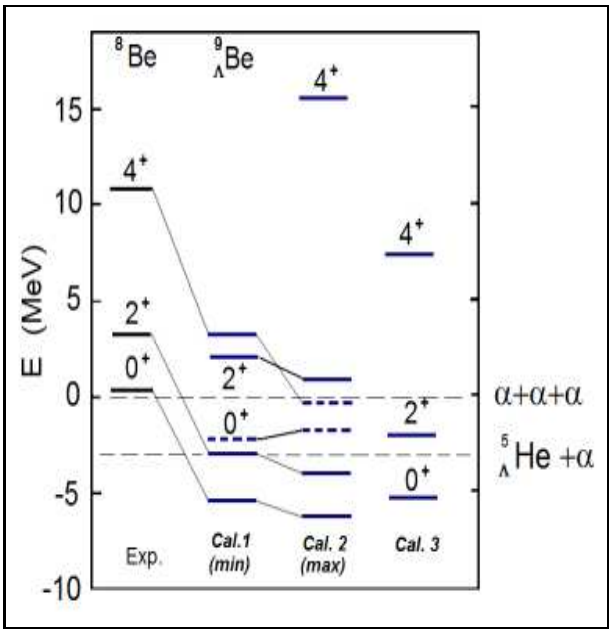

Figure 2. Experimental data [13] for energies of ^{8}Be and ${}^9_\Lambda$Be ground band levels calculated with the Gibson $\alpha\Lambda$ potential. $Cal.1(min)$ corresponds to the results of the calculations with "minimal" configuration of orbital momenta. For $Cal.2(max)$ the orbital momenta are taken in "maximal" configuration. $Cal.3$ corresponds to the results of [6]. Virtual states are shown by dashed lines.

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